The New Quantitative Trade Model: Equilibrium and Welfare Analysis

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Motivation

- **Quantitative trade models:**
  - Multiple sectors and intermediates
  - Roundabout production matching IO data
  - Caliendo-Parro model, CRS

- **Should extend to allow for EES:**
  - New trade theory: EES due to love of variety
  - Empirical evidence for EES (recent): Costinot et. al. ’19; BCDR; Lashkaripour and Lugovskyy ’22; Bartelme et. al. ’23; Breinlich et. al. ’21
Examples in the Literature

- **BCDR '21**: industrial policy (PC)
- **Bartelme et. al. '23**: trade shocks on growth (PC)
- **Lashkaripour and Lugovskyy '22**: industrial policy (MC)
- **Breinlich et. al. '21**: import shocks on exports (MC)

**Special cases:**
- **Krugman and Venables '95**: core-periphery
- **Antras et. al. '21**: trade policy
- **Caliendo et. al. '21**: optimal trade policy (Melitz)
- **Baqee and Farhi '21**: local comparative statics (no trade)

**Background:**
- **KLR**: multi-sector gravity + EES, no intermediates
This Paper

- **Model:**
  - Caliendo-Parro + EES in VA or GO, Small Open Economy
  - $\varepsilon_k$ is the trade elasticity, $\theta_k$ is the scale elasticity

- **Uniqueness:**
  - Sufficient *Uniqueness Condition (UC):*
    \[ \sum_s \theta_s \ell_{sk} \varepsilon_k < 1 \text{ for all } k \]
  - Without IO: $\ell_{kk} = 1$ and $\ell_{sk} = 0$ for $s \neq k \Rightarrow$ KLR’s condition:
    \[ \theta_k \varepsilon_k < 1 \text{ for all } k \]
  - **NOTE:** Proof is not yet complete for EES in GO

- **Gains from Trade:**
  - With EES in VA, UC implies gains from trade
  - With EES in GO, could have losses from trade even under UC
1. Motivation

2. Model
   2.1 Basic Assumptions
   2.2 Equilibrium

3. Characterization of Equilibrium

4. Gains from Trade
Basic Assumptions

- Home is SOE
- $K$ sectors indexed by $k = 1, \ldots, K$
- Armington assumption
- Perfect competition and sector-level EES
Basic Assumptions

\[ Q_k = \left( \alpha_k - \alpha_k \prod_{s=1}^{K} \alpha_{sk}^{\alpha_k} \right) \overline{T}_k L_k^{\alpha_k} \prod_{s=1}^{K} Q_{sk}^{\alpha_{sk}} \]

\[ \alpha_{sk} \in [0, 1], \quad \alpha_k + \sum_s \alpha_{sk} = 1, \quad \text{and} \quad \alpha_k > 0 \]

\[ \overline{T}_k = T_k L_k^{\alpha_k \gamma_k} \prod_{s=1}^{K} Q_{sk}^{\alpha_{sk} \nu_k} \]

\[ \nu_k < \frac{\alpha_k}{1 - \alpha_k} \]
Basic Assumptions

- If \( \nu_k = 0 \), then

\[
Q_k = \left( \alpha_k^{-\alpha_k} \prod_{s=1}^{K} \alpha_{sk}^{-\alpha_{sk}} \right)^{T_k} \cdot \left( L_k \cdot L_k^{\gamma_k} \right)^{\alpha_k} \prod_{s=1}^{K} Q_{sk}^{\alpha_{sk}}
\]

- This is EES in VA, a natural framework for technological EES

- If \( \gamma_k = \nu_k \), then

\[
\bar{T}_k = \tilde{T}_k \cdot Q_k^{\gamma_k \over 1+\gamma_k}
\]

- This is EES in gross output, and results from Krugman with

\[
\gamma_k = \nu_k = {1 \over \sigma_k - 1}
\]

where \( \sigma_s \) is the EoS across domestic varieties
Basic Assumptions

- Composite consumption ≠ composite intermediate

\[
\lambda^C_k(p_k) = \frac{p_k^{-\varepsilon_k}}{p_k^{-\varepsilon_k} + [p^C_*]^{-\varepsilon_k}}, \quad \lambda^I_k(p_k) = \frac{p_k^{-\varepsilon_k}}{p_k^{-\varepsilon_k} + [p^I_*]^{-\varepsilon_k}}
\]

- Cobb-Douglas preferences across sectors

\[
C_k = \lambda^C_k(p_k)e_k w\bar{L}
\]

- Isoelastic export revenues in sector \( k \)

\[
X_k = E_k p_k^{-\varepsilon_k}
\]
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Equilibrium: Prices

- Equilibrium prices given $\lambda^I_1, \ldots, \lambda^I_K$ and $L_1, \ldots, L_K$:

$$p_k = \xi_k \cdot w \cdot \prod_s \left[ \frac{\ell^F_{sk} - \delta_{sk}}{\varepsilon_s} \right] \cdot \prod_s L_s^{-\theta_s \ell^F_{sk}},$$

where $\delta_{sk}$ indicator function for $s = k$,

$$\theta_s \equiv \alpha_s \gamma_s + (1 - \alpha_s) \upsilon_s$$

and

$$L^F \equiv (I - AD \upsilon)^{-1} \quad \text{with} \quad A \equiv \{ \alpha_{sk} \}, \quad D \upsilon \equiv D \{ \upsilon \}$$

capture forward linkages,

$$\ell^F_{sk} = -\partial \ln p_k / \partial \ln T_s$$
Equilibrium: Market Clearing

Market clearing condition in sector $k$ is

$$p_k Q_k = C_k + X_k + \lambda_k^{I} \sum_s P_k Q_{ks}$$

or

$$L_k/\alpha_k = d_k + \lambda_k^{I} \sum_s \alpha_{ks} L_s/\alpha_s$$

Solving for $L_k$,

$$R_k \equiv L_k/\alpha_k = \sum_s \tilde{l}_{ks}^B d_s$$

where

$$\tilde{L}^B \equiv (I - D\lambda A)^{-1} \quad \text{with} \quad D\lambda \equiv \mathcal{D}\{\lambda^I\}$$

captures backward linkages, $\tilde{l}_{ks}^B = \partial R_k/\partial d_s$ (holding trade shares fixed)
An equilibrium is a wage \( w \), prices \( p \) and labor allocations \( L \) that satisfy

\[
p_k = \xi_k \cdot w \cdot \prod_s \left[ \lambda^l_s(p_s) \right]^{\ell^F_{sk} - \delta_{sk}} \cdot \prod_s L_s^{-\theta_s \ell^F_{sk}}
\]

\[
L_k/\alpha_k = d_k(w, p_k) + \lambda^l_k(p_k) \sum_s \alpha_{ks} L_s / \alpha_s
\]

\[
\sum_k L_k = \bar{L}
\]

We next *show* that there is a unique solution if

\[
\sum_s \theta_s \ell^F_{sk} \varepsilon_k < 1 \quad \text{for all } k \quad \text{(UC)}
\]
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Characterization of Equilibrium

- **Step 1**: Take $w$ and $L$ as given and focus on $p$:
  
  Show that $(UC) \implies$ There exists a unique $p$

  - This leads to function $p(w, L)$

- **Step 2**: Take $w$ as given, focus on $L$:
  
  Show that $(UC) \implies$ There exists a unique $L$

  - This leads to labor demand $L(w)$

- **Step 3**: Focus on $w$:
  
  Show that $(UC) \implies$ There exists a unique $w$
Characterization of Equilibrium

Steps 1 and 3 are straightforward, step 2 is challenging

- The goods market clearing condition gives a mapping \( L \rightarrow L' \),

\[
L'_k / \alpha_k = d_k (p_k (L)) + \lambda_k^l (p_k (L)) \sum_s \alpha_{ks} L'_s / \alpha_s
\]

- Existence is proved by showing that (given UC) this mapping stays inside a rectangular region of \( \mathbb{R}^K_{++} \)

- To show uniqueness we use the “Index Theorem”
Index at a fixed point is $+1$ ($-1$) if $1 - F'(L) > 0$ ($< 0$)

Generalization: index is $\text{sgn} \left( \det (I - J) \right)$

Index Theorem: sum of indices $= +1$
Index Theorem

- **Key implication:**
  
  \[ \det(I - J) > 0 \text{ at any fixed point} \implies \text{fixed point is unique} \]

- **Basic idea:** if a self-absorbing mapping is a *local* contraction mapping at each fixed point, then it has only one fixed point.

- **Economics in our application:** supply curve cuts demand curve from below at every goods market equilibrium.
Jacobian

- **UC** $\iff$ $\det(I - J) > 0$ or $\rho(J) < 1$ for $J =$ Jacobian of $L \to L'$ (in logs) mapping at a fixed point

- With no trade in intermediates,

$$J|_{D\lambda}=I = D_R^{-1} \cdot L^B \cdot \left\{ \frac{\partial d_k}{\partial \ln p_r^{\varepsilon_k}} \right\} \cdot \left\{ \frac{\partial \ln p_k^{\varepsilon_k}}{\partial \ln L_s} \right\}$$

$$\leq \mathcal{D} \left\{ L^B D_d l \right\}^{-1} \cdot L^B \cdot D_d \cdot D_{\varepsilon} \left[ L^F \right]^T D_{\theta} \equiv \tilde{J}|_{D\lambda}=I$$

- Stochastic matrix

- Thus $\rho \left( \tilde{J}|_{D\lambda}=I \right) < 1$ if max row sums of $D_{\varepsilon} \left[ L^F \right]^T D_{\theta}$ are $< 1$, which is our UC,

$$\sum_s \theta_s \epsilon_{sk}^{\varepsilon_k} < 1, \forall k$$
Uniqueness Condition

- In the case with EES in VA we show that the UC

\[ \sum_s \theta_s \ell^F_{sk} \varepsilon_k < 1, \forall k \]

is sufficient for \( \rho(J) < 1 \) for any \( \lambda^I \)

- **Intuition:** works with autarky in intermediates and 100% export demand, where strength of linkages and elasticity of demand are maximized
  - **Wrinkle:** NTS that rate of change in linkages is controlled by UC

- **Necessity:** UC is weakest condition that works for all supply/demand shifters

- Still working on this proof with EES in GO
With EES in VA ($\nu_k = 0$, $\forall k$) and $\gamma_k = \gamma$, $\forall k$:

\[ \nu_k = 0, \forall k \implies \theta_k = \gamma \alpha_k \text{ and } \ell_{sk}^F = \ell_{sk}^B, \forall s, k \text{ so UC becomes } \]

\[ \gamma \sum_s \alpha_s \ell_{sk}^B \varepsilon_k = \gamma \varepsilon < 1, \forall k, \]

where we have used $\sum_s \alpha_s \ell_{sk}^B = 1$

This is same UC in KLR for case without IO if $\gamma_k = \gamma, \forall k$

With EES in GO ($\nu_k = \gamma_k$, $\forall k$) and $\gamma_k = \gamma$, $\forall k$ then UC becomes

\[ \varepsilon_k \leq \frac{1}{\gamma \sum_s \ell_{sk}^F} \]

EES in GO leads to increased amplification relative to EES in VA
\( \varepsilon_k \leq \frac{1}{\gamma \max_k \sum_s \ell^F_{sk}} \) for US (Motor Vehicles)
<table>
<thead>
<tr>
<th>Sector</th>
<th>Max $\epsilon_k$ (1)</th>
<th>Max $\epsilon_k$, 10$^{th}$ pctile (2)</th>
<th>Max $\epsilon_k$, US (3)</th>
<th>Max $\epsilon_k$, Avg. IO (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>2.8</td>
<td>3.3</td>
<td>4.2</td>
<td>5.3</td>
</tr>
<tr>
<td>Mining</td>
<td>3</td>
<td>3.5</td>
<td>5</td>
<td>6.1</td>
</tr>
<tr>
<td>Textiles</td>
<td>2.2</td>
<td>2.7</td>
<td>4.4</td>
<td>3.5</td>
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<tr>
<td>Chemical Products</td>
<td>2.1</td>
<td>2.6</td>
<td>3.9</td>
<td>3.5</td>
</tr>
<tr>
<td>Basic Metals</td>
<td>1.8</td>
<td>2.3</td>
<td>3.2</td>
<td>3.1</td>
</tr>
<tr>
<td>Machinery &amp; Equipment</td>
<td>1.9</td>
<td>2.6</td>
<td>3.8</td>
<td>3.4</td>
</tr>
<tr>
<td>Motor Vehicles</td>
<td>1.9</td>
<td>2.2</td>
<td>2.9</td>
<td>2.9</td>
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<tr>
<td>Construction</td>
<td>2.3</td>
<td>2.7</td>
<td>4.5</td>
<td>4</td>
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<tr>
<td>Wholesale/Retail Trade</td>
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<td>3.9</td>
<td>5.8</td>
<td>5.5</td>
</tr>
<tr>
<td>Finance &amp; Insurance</td>
<td>2.4</td>
<td>4.4</td>
<td>4.9</td>
<td>5.3</td>
</tr>
<tr>
<td>Education</td>
<td>3.9</td>
<td>5.1</td>
<td>6.5</td>
<td>6.6</td>
</tr>
<tr>
<td>Avg. Ratio w/ column 1</td>
<td>1</td>
<td>0.8</td>
<td>0.5</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Assumes $\gamma_k = \nu_k = 0.1$, $\forall k$. 
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Gains from Trade

To simplify, use $\lambda_k^C = \lambda_k^I = \lambda_k$. We then have

$$GT = \prod_k (\lambda_k)^{-\psi_k^F} \times \prod_k \left( \frac{L_k}{\alpha_k \psi_k^B \bar{L}} \right)^{\theta_k \psi_k^F}$$

where $\psi_k^F \equiv \sum_s \ell_{ks}^F e_s$ and $\psi_k^B \equiv \sum_s \ell_{ks}^B e_s$ are (closed economy) forward and backward Domar weights (Baqee and Farhi ’21)

Combined with

$$wL_k = \alpha_k \sum_s \tilde{\ell}_{ks}^B (\lambda_s e_s w \bar{L} + X_s) \geq \alpha_k \sum_s \tilde{\ell}_{ks}^B \lambda_s e_s w \bar{L}$$

we then have

$$GT \geq GT^* (\lambda) \equiv \prod_k \lambda_k^{\frac{\psi_k^F}{\varepsilon_k}} \times \prod_k \left( \frac{\sum_r \tilde{\ell}_{kr}^B (\lambda) e_r \lambda_r}{\psi_k^B} \right)^{\theta_k \psi_k^F}$$
Gains from Trade

- KLR showed that

\[ \gamma_k \varepsilon_k < 1 \implies P_k \downarrow \text{ as } \lambda_k \downarrow \text{ below one } \implies GT^* > 1 \]

- Condition \( \gamma_k \varepsilon_k < 1 \) also guarantees uniqueness

- Does UC also guarantee \( GT^* > 1 \) in the current setting?
We show that $GT^* (\lambda)$ is strictly (log-log) convex in $\lambda$ so if

$$
- \frac{\partial \ln GT^*}{\partial \ln \lambda_i} \bigg|_{\text{Autky}} \geq 0
$$

for all $i$ then $GT^* > 1$ for any trade pattern.
Gains at Autarky

We have

\[- \frac{\partial \ln G_T^*}{\partial \lambda_i} \bigg|_{\text{Autky}} = \frac{\psi_i^F}{\varepsilon_i} - \sum_k \psi_k^F \theta_k \frac{\partial \ln L_k^*}{\partial \lambda_i} \bigg|_{\text{Autky}}\]

and

\[\frac{\partial \ln L_k^*}{\partial \lambda_i} \bigg|_{\text{Autky}} = \frac{\ell_{ki}^B \psi_i^B}{\psi_k^B}.\]

Using \(\Psi_k \equiv \psi_k^F / \psi_k^B\) for “distortion centrality of sector \(k\)” (Liu ’19) and \(m_i \equiv (1 - \lambda_i) \left[ e_i + \sum_s \alpha_{is} R_s / \bar{L} \right]\) for imports in sector \(i\) as a share of GDP, we have

\[- \frac{\partial \ln G_T^*}{\partial m_i} \bigg|_{\text{Autky}} = \frac{\psi_i}{\varepsilon_i} - \sum_k \theta_k \Psi_k \ell_{ki}^B.\]
Gains at Autarky: EES in VA

\[
\left. \frac{\partial \ln GT^*}{\partial m_i} \right|_{\text{Autky}} = \frac{\Psi_i}{\varepsilon_i} - \sum_k \theta_k \Psi_k \ell^B_{ki}
\]

\[
\left. \frac{\partial \ln GT^*}{\partial m_i} \right|_{\text{Autky}} = \frac{1}{\varepsilon_i} - \sum_k \theta_k \ell_{ki} > 0, \forall i \implies GT^* > 1
\]

If EES in VA then \( L^B = L^F \) and so \( \psi^F_k = \psi^B_k, \forall k \) plus the UC \( \implies \)

\[
\left. \frac{\partial \ln GT^*}{\partial m_i} \right|_{\text{Autky}} = \frac{1}{\varepsilon_i} - \sum_k \theta_k \ell_{ki} > 0, \forall i \implies GT^* > 1
\]
Gains at Autarky: EES in GO

\[
\frac{\partial \ln GT^*}{\partial m_i} \bigg|_{\text{Autky}} = \frac{\Psi_i}{\varepsilon_i} - \sum_k \theta_k \psi_k \ell^B_{ki}
\]

**If** EES in GO then \(\mathcal{L}^B \neq \mathcal{L}^F\) so UC can hold while \(-\frac{\partial \ln GT^*}{\partial m_i} \bigg|_{\text{Autky}} < 0\)

**Assuming** \(\varepsilon_i = \varepsilon, \forall i\) and \(\theta_k = \theta, \forall k\) then

\[
\frac{\partial \ln GT^*}{\partial m_i} \bigg|_{\text{Autky}} = \frac{1}{\varepsilon} \frac{\Psi_i}{\text{Dist. Centrality}} - \theta \sum_k \psi_k \ell^B_{ki} \quad \text{Backward Dist. Centrality}
\]

**Conclusion:** imports in sectors with low distortion centrality but high backward distortion centrality can cause welfare losses
Adding Exports

- To a first order, the gains from trade at autarky are

\[ \ln GT \approx \sum_k \frac{\Psi_k}{\varepsilon_k} m_k + \sum_{k,s} \theta_k \Psi_k \ell^B_{ks} (x_s - m_s), \]

where \( m_s \) and \( x_s \) are imports and exports relative to sectoral GDP.

- If EES in VA then

\[ \ln GT \approx \sum_k \frac{m_k}{\varepsilon_k} + \sum_s \bar{\gamma}_s (x_s - m_s), \]

where \( \bar{\gamma}_s \equiv \sum_k \gamma_k \alpha_k \ell^B_{ks} \)

- Higher gains if specialize in sectors with high backward EES.
Adding Exports

- To a first order, the gains from trade at autarky are

\[
\ln GT \approx \sum_k \frac{\Psi_k}{\varepsilon_k} m_k + \sum_k \theta_k \Psi_k \ell_{ks}^B (x_s - m_s),
\]

where \( m_s \) and \( x_s \) are imports and exports relative to sectoral GDP.

- If EES in GO and common elasticities then

\[
\ln GT \approx \frac{1}{\varepsilon} \sum_k m_k + \frac{1}{\varepsilon} \sum_k (\Psi_k - 1) m_k + \theta \sum_{k,s} \Psi_k \ell_{ks}^B (x_s - m_s)
\]

- Second term higher if imports mostly in high \( \Psi \) sectors, which tend to be upstream (Liu '19)
- Third term higher if specialize in sectors with high backward distortion centrality
Conclusions

- Incorporate EES into quantitative trade models
- Open computational black box: equilibrium and welfare properties
- Sufficient condition for uniqueness

\[ \sum_s \theta_s \ell_{sk}^F \varepsilon_k < 1 \text{ for all } k \]

- Nests simpler condition \( \theta_k \varepsilon_k < 1 \) without IO
- IO makes upper bound on \( \theta' \)'s much tighter

- UC ensures gains if EES in VA, but not if EES in GO